Route Towards a Formal Written Method for Division – Year 1

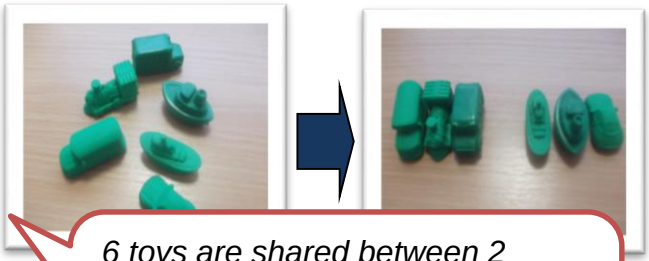
National Curriculum Programme of Study;

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

BY THE END OF YEAR 1…

Understanding division as sharing and grouping.
Recognise half as one of two equal parts of a quantity.
Recognise quarter as one of four equal parts of a quantity.

Understanding both 'equal sharing' and 'grouping'

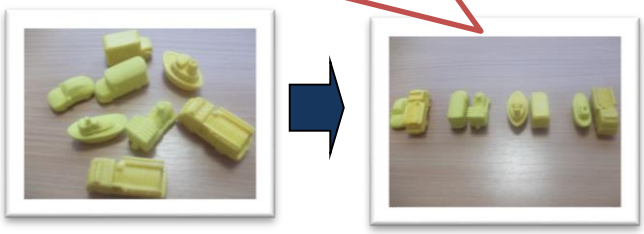


6 toys are shared between 2 children. How many will they have each? …3
They have half each.Half of 6 is 3.

Equal sharing occurs when a quantity is shared out equally into a given number of portions, and we work out how many there are in each portion.

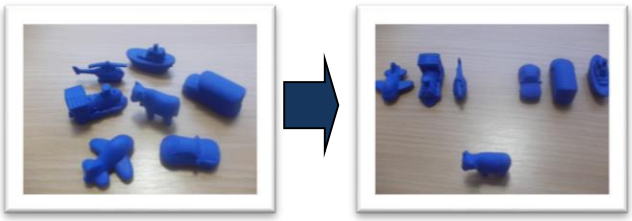
When we share we know how many we have to share out and how many to share between but not how many they will each get.

There are 8 toys grouped into 4. Each child can have 2 toys. How many children will get 2 toys?
They have a quarter of the toys. A quarter of 8 is 2.



Grouping occurs when we are asked to find how many groups of the divisor are in the original amount.
We know how many we have and how many to put into each 'set' but not the number of 'sets' we will have.

Introducing remainders when dividing



7 toys are shared between 2 children. How many will they have each?
They have 3 each with one left over or remainder of 1.

Left Over (Remainder) occurs when a group cannot be shared equally. Introduce the concept of remainder to the children, using 'everyday' objects and real life contexts where possible.

Route Towards a Formal Written Method for Division – Year 2

National Curriculum Programme of Study;

- recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

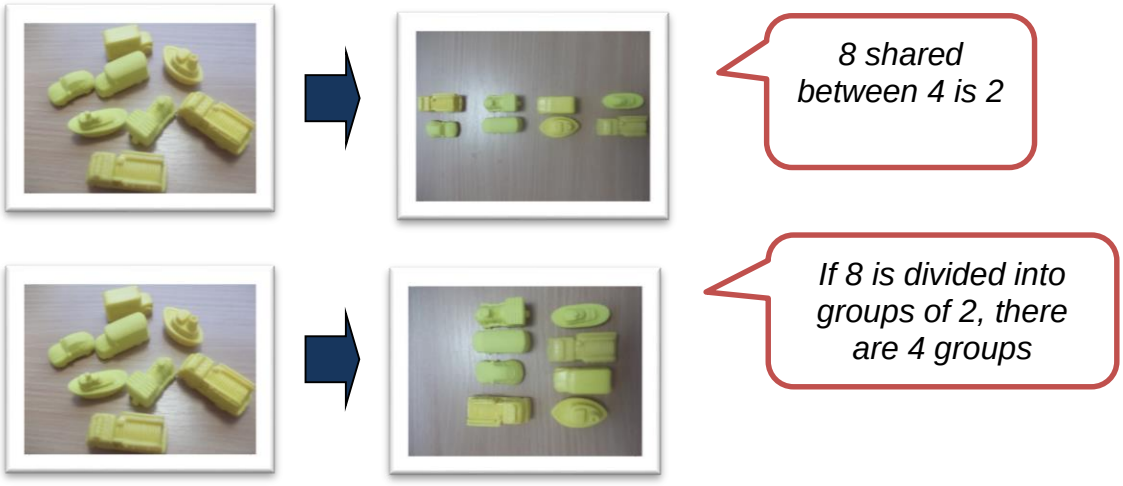
BY THE END OF YEAR 2…

By the end of Year 2, children will be able to show their understanding as;

Arrays showing understanding of grouping in 2s, 5s and 10s,
e.g. 12 ÷ 5 = 2 remainder 2

Introducing arrays for visualisation of division

Sharing and grouping should be formalised into arrays introducing the vocabulary of mathematical statements.



When the children can discuss their division confidently, using the language of both sharing and grouping accurately, the mathematical signs should be introduced for accompanying number sentences, e.g. 8 ÷ 4 = 2 and 8 ÷ 2 = 4

When dividing by 2, use this as an opportunity to link with fractions work.
E.g. 8 ÷ 2…. *How can we read this? How many groups of 2 are there in 8? If I share 8 between 2 people, how many would they each get? What is one half of 2?*

Links should also be made to multiplication work, e.g. 2 x 4 = 8, 4 x 2 = 8

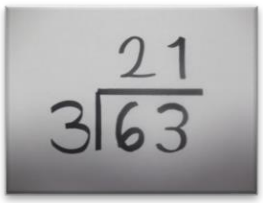
Route Towards a Formal Written Method for Division – Year 3

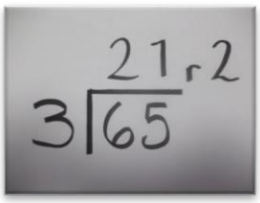
National Curriculum Programme of Study;

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

BY THE END OF YEAR 3…

By the end of Year 3, children will be able to show their understanding as;





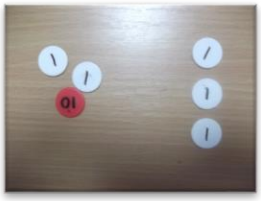
Short written method for division, with no exchange required

63 ÷ 3 = 21

65 ÷ 3 = 21 r 2

Dividing by a single digit divisor

Use of place value counters to divide by a single digit number. Include examples where pupils need to exchange tens for ones.

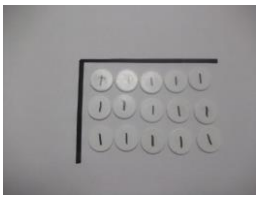


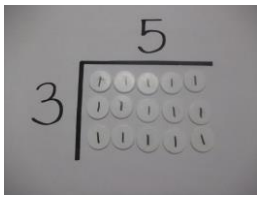
E.g. 15 ÷ 3
Start by representing the dividend (15) in the smallest number of counters possible.
In this example, 1 ten-counter and 5 one-counters.

Explain that the counters need to be shared equally between three rows, or into thirds. Start with the most significant digit and discuss the difficulty of sharing the single ten-counter. Suggest that the counter is exchanged for 10 one-counters. The 15 one-counters can then be shared into three equal rows, or thirds. Indicate the rows, or thirds to the children, as well as the groups of 3 counters.

Visual representation of the standard short division written method

The place value counters can continue to be used to provide a visual representation of the standard short division method.





This layout is based on the above method, making strong links to the use of arrays for division as well as multiplication.

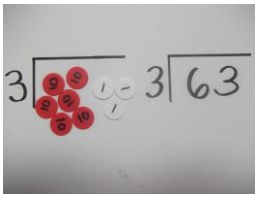
The boundary line is added to introduce children to the convention used when writing short division calculations.

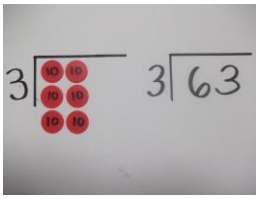
Ensure the children are talking about their calculations using accurate mathematical vocabulary,

e.g. "15 counters shared between three equals 5", "there are five groups of 3 in 15", and *"one third of 15 equals 3"*

Using place value counters to explain the short division written method

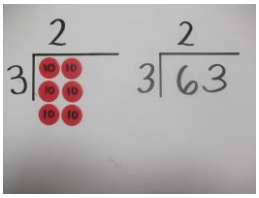
The place value counters can provide a useful vehicle for exposing the structure of the mathematics behind the 'short division' method. Counters should be used and the written method completed alongside, simultaneously. As the written method progresses, children can 'see' the mathematics in the physical nature of the counters
E.g. 63 ÷ 3



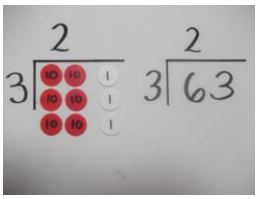


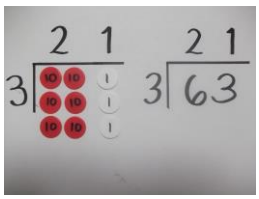
The smallest number of counters are used; 6 tens and three ones.

The boundary line is drawn and the tens counters are shared equally among the three rows, or thirds.



Each of the three rows received 2 tens counters, or 20. This is indicated above the boundary line, in the tens column.





The three ones-counters are then shared equally between the three rows, resulting in a single one-counter in each row.
The corresponding annotation is added above the boundary line.

Children in Year 3 should also be provided with examples where there will be a remainder. The visual model of a counter being 'left over' provides a useful starting point for a discussion about remainders and the context in which the question is set.

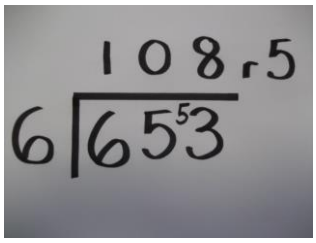
Route Towards a Formal Written Method for Division – Year 4

National Curriculum Programme of Study;

- recall multiplication and division facts for multiplication tables up to 12 × 12
- Divide two-digit and three-digit numbers by a one-digit number using formal written layout

BY THE END OF YEAR 4…

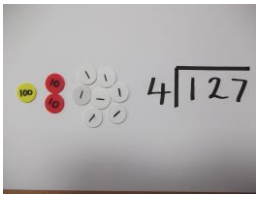
By the end of Year 4, children will be able to show their understanding as;



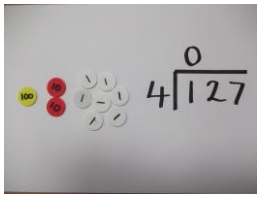
Short written method of division of 3-digit numbers by 1-digit numbers. Examples will include the need to exchange as well as remainders.

E.g. 653 ÷ 6

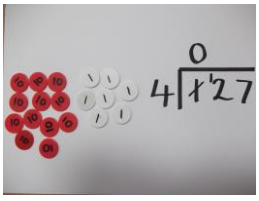
Using place value counters to explain the short division method (requiring exchange)



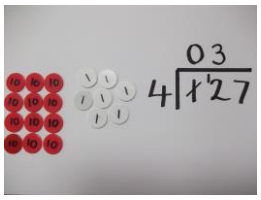
127 ÷ 4
The dividend is represented in the smallest number of counters possible



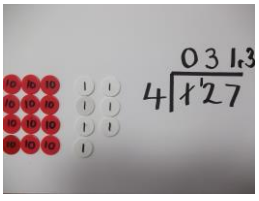
The single 100-counter cannot be divided equally between the four rows, so a zero is written in the hundreds column.



The single 100-counter is exchanged for 10 ten-counters. Allow the children time to add up the counters to reinforce that they still represent the same value. There are now an additional 10 ten-counters. Demonstrate how this is recorded; crossing out the exchanged 100-counter and showing that there are now 12 ten-counters.



The 12 ten-counters can be shared equally between the four rows (or quarters), enabling each to have three counters, or 30. This is annotated in the tens column.



The 7 remaining one-counters can be shared between the four rows, resulting in each receiving 1 counter, and three one-counters remaining. The annotation should be added alongside the physical representation.

When children are confidently using the correct language to explain their conceptual understanding of the division calculation using the place value counters, they may choose to simply use the written form, often 'imagining' the counters in their head.

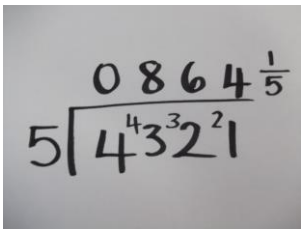
Route Towards a Formal Written Method for Division – Year 5

National Curriculum Programme of Study;

- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

BY THE END OF YEAR 5…

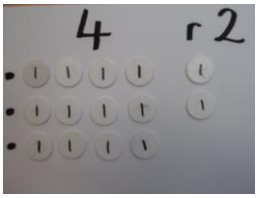
By the end of Year 5, children will be able to show their understanding as;



Dividing a 4 digit number by 1 digit with an appropriate remainder (fraction)

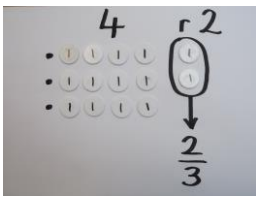
E.g. 4321 ÷ 5

Using place value counters to represent remainders as fractions



Start with a simple example, e.g. 14 ÷3, arranging the counters into 3 rows, or groups of three, as before. Discuss the 2 counters remaining and how this is expressed.

Provide children with a context where 'remainder 2' is not appropriate, where any remainder also needs to be shared, such as dividing 14 cakes between 3 people.



Model how the two remaining counters are two of the next group of three, effectively two thirds of the next group. So when looking at how many groups of three could be made, the result is 4 and 2/3.

If the two remaining counters were shared equally between the three rows, each row would receive an additional 2/3 of a counter.

Route Towards a Formal Written Method for Division – Year 6

National Curriculum Programme of Study;

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

BY THE END OF YEAR 6…

By the end of Year 6, children will be able to show their understanding as;

Divide 4 digit by 2 digit using 'multiple of the divisor method' (chunking)
Divide 4 digit by 2 digit using long division method.
Divide 4 digit by 2 digit using short division method

Long division using multiples of the divisor (HTU ÷ TU)

```
        0  2  8
    ┌──────────
15  │ 4  3  2
    │ 3  0  0      15 X 20
    ├──────────
    │ 1  3  2
    │ 1  2  0      15 X 8
    ├──────────
    │    1  2
```

$$\frac{12}{15} = \frac{4}{5}$$

$$= 28\ {}^{4}/_{5}$$