

Route Towards a Formal Written Method for Multiplication – Year 1

National Curriculum Programme of Study;

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

BY THE END OF YEAR 1...

Children will be able to understand multiplication as describing an array.

Arranging objects into groups of the same size, with support

With support, children in year 1 should be arranging a variety of physical objects into groups of the same size, counting the number of groups, the amount in each group, and the total.



12 green cotton reels arranged into groups of 3



6 clothes pegs arranged into groups of 2

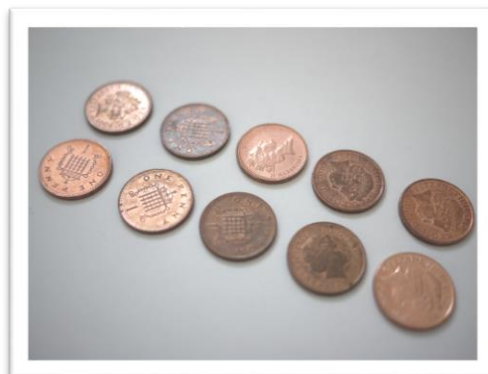


8 socks arranged into groups of 2, or pairs

Arranging objects into rectangular arrays

To support the progression towards a formal visual image of multiplication, and into a formal written method in Key Stage 2, children need to be shown how to arrange their objects into a rectangular array.

Children in year 1 will be counting in steps of 2, 5 and 10, and so it is useful if these numbers are used initially in any arrays created.



Route Towards a Formal Written Method for Multiplication – Year 2

National Curriculum Programme of Study;

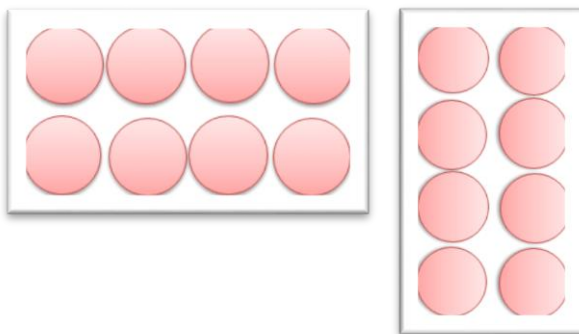
- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs
- show that multiplication of two numbers can be done in any order (commutative).
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

BY THE END OF YEAR 2...

Children should be able to recognise arrays in printed form and in the environment, and be able to describe them in terms of repeated addition as well as with a multiplication statement.

Following on from year 1...

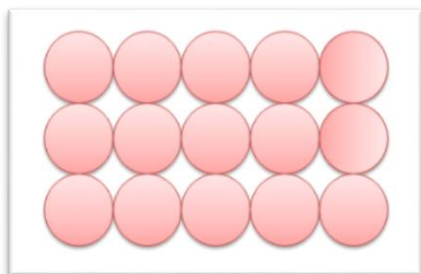
Making arrays using physical objects and representing them in drawings



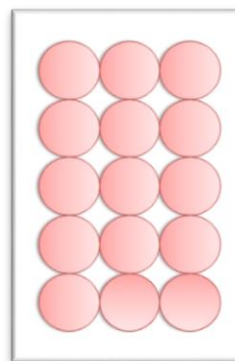
Children should be familiar with the array image for multiplication from their experiences in Year 1. Children in year 2 should continue their experiences with arrays, arranging counters, drawing their own representations, counting and labelling the groups.

E.g. 2 groups of 4 (starting from the top of the array or 4 groups of 2 (starting from the top of the array).

Annotating arrays using repeated addition



array
E.g. 5



Children should be encouraged to see the array as a number of counters repeated in rows.
 $+ 5 + 5 = 15$

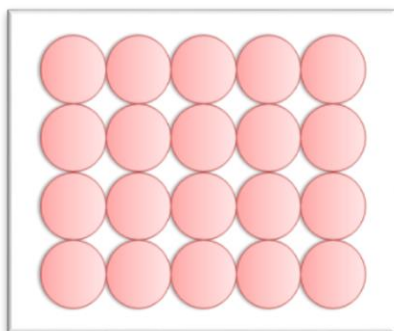
$$3 + 3 + 3 + 3 + 3 = 15$$

N.B.

These examples encourage children to see an array as being built up from left to right, but practice needs to be consistent.

array being built up from the top row.

Annotating arrays using multiplication



$$5 + 5 + 5 + 5 = 20$$
$$5 \times 4 = 20$$

This annotation links to the idea of multiplication as 'scaling'; making a number so many times bigger. Here the starting number is 5 and it is 'scaled up' four times. Many teachers refer to multiplication as meaning 'lots of'. If this is consistent practice in your school, then this diagram would represent 4 'lots of' 5, or 4×5 .

Route Towards a Formal Written Method for Multiplication – Year 3

National Curriculum Programme of Study;

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

BY THE END OF YEAR 3...

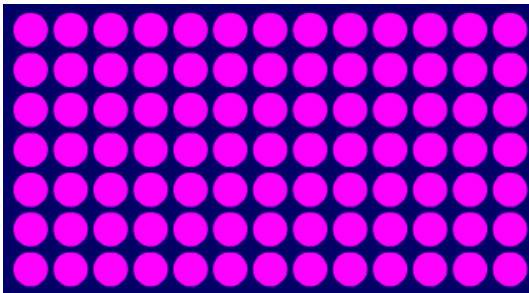
$$\begin{array}{r}
 36 \\
 \times 7 \\
 \hline
 42 \\
 \hline
 210 \\
 \hline
 252
 \end{array}$$

Expanded column written method,
progressing to the compact column
written method

$$\begin{array}{r}
 36 \\
 \times 7 \\
 \hline
 252 \\
 \hline
 4
 \end{array}$$

Following on from year 2...

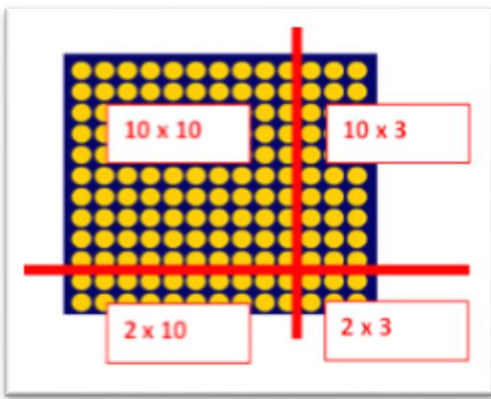
Using arrays and known facts for multiplying a two-digit by a single-digit number
and then a two-digit by a two-digit number



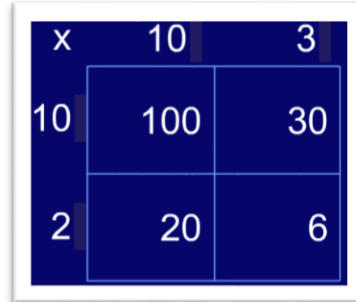
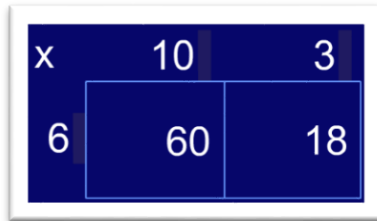
$$\begin{aligned}
 \text{E.g. } 7 \times 13 &= (5 \times 10) + (5 \times 3) + (2 \times 10) + (2 \times 3) \\
 &= 50 + 15 + 20 + 6 \\
 &= 91
 \end{aligned}$$

5 x 3

2 x 3



$$\begin{aligned}
 \text{E.g. } 12 \times 13 &= (10 \times 10) + (10 \times 3) + (2 \times 10) + (2 \times 3) \\
 &= 100 + 30 + 20 + 6 \\
 &= 156
 \end{aligned}$$



The use of the array develops the required understanding for the grid method. Initially the grid should be introduced alongside the array, enabling the children to discuss what is the same and what is different.

Expanded column method for multiplication (TU x U)

$ \begin{array}{r} 36 \\ \times 7 \\ \hline 42 \\ 210 \\ \hline 252 \end{array} $	<p>The expanded column method for multiplication should only be introduced once children are secure with the grid method shown above. They should be able to explain the structure of the grid and how it helps to find the answer to the multiplication calculation.</p>
$ \begin{array}{r} 210 \\ 252 \\ \hline 252 \end{array} $	<p>The column notation should be shown alongside the same example represented as a grid (and even an array), enabling the children to see the similarities. Model the expanded column method, paying particular attention to the value of the digits involved, and showing where the same part appears in the grid method.</p>

Route Towards a Formal Written Method for Multiplication – Year 4

National Curriculum Programme of Study;

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

BY THE END OF YEAR 4...

By the end of Year 4, children will be able to show their understanding as;

Compact column written method for multiplication

$$\begin{array}{r}
 342 \\
 \times \quad 7 \\
 \hline
 2394 \\
 2
 \end{array}$$

Following on from Year 3...

Expanded column method for multiplication (TU x U)

x	30	6
7	210	42

$$\begin{array}{r}
 36 \\
 \times 7 \\
 \hline
 42 \\
 210 \\
 \hline
 252
 \end{array}$$

The expanded column method for multiplication should only be introduced once children are secure with the grid method detailed in Year 3. They should be able to explain the structure of the grid and how it helps to find the answer to the multiplication calculation.

The column notation should be shown alongside the same example represented as a grid (and even an array), enabling the children to see the similarities. Model the expanded column method, paying particular attention to the value of the digits involved, and showing where the same part appears in the grid method.

Compact column method for multiplication (TU x U)

Once confident with the expanded column method for multiplication, and showing considerable conceptual understanding, children can progress towards the compact method.

As at all earlier stages, this should be introduced alongside the previous expanded method, enabling children to understand the positioning of numbers, reducing the need to teach a 'process'.

$$\begin{array}{r}
 36 \\
 \times 7 \\
 \hline
 252 \\
 4
 \end{array}$$

Route Towards a Formal Written Method for Multiplication – Year 5

National Curriculum Programme of Study;

- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

BY THE END OF YEAR 5...

By the end of Year 5, children will be able to show their understanding as;

$\begin{array}{r} 2741 \\ \times \\ \hline 16446 \\ \hline 42 \end{array}$	$\begin{array}{r} 4276 \\ \times 34 \\ \hline 17104 \\ 128280 \\ \hline 145384 \\ \hline 1 \end{array}$
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Following on from Year 4...

Formal column method for short multiplication (HTU x U)

$$\begin{array}{r} 143 \\ \times 6 \\ \hline 858 \\ \hline 21 \end{array}$$

The compact column method for multiplication of HTU x U is introduced towards the end of Year 4, once children have worked through the grid and expanded stages to embed conceptual understanding.

Consolidation of short multiplication should continue with increasingly large numbers, using the same method as that taught previously. The grid or expanded methods should be returned to as appropriate.

$$\begin{array}{r} 2741 \\ \times 6 \\ \hline 16446 \\ \hline 42 \end{array}$$

Short multiplication involving decimal numbers

Decimal multiplication should be introduced in context, e.g. requiring children to calculate the cost of 6 items, priced at £3.25 each.

The expanded method should be used for recording, ensuring that the accompanying explanation details the value of the digits in terms of their monetary value;

'6 lots of 5 pence are 30 pence... 6 lots of 20 pence is £1.20... 6 lots of £3 is £18.00... 30 pence add 20 pence is 50 pence, £18 add £1 is £19... giving a total of £19.50

$$\begin{array}{r} 3.25 \\ \times 6 \\ \hline .30 \\ 1.20 \\ 18.00 \\ \hline 19.50 \end{array}$$

Route Towards a Formal Written Method for Multiplication – Year 6

National Curriculum Programme of Study;

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- multiply one-digit numbers with up to 2 decimal places by whole numbers

BY THE END OF YEAR 6...

By the end of Year 6, children will be able to show their understanding as;

$$\begin{array}{r}
 4 \ 2 \ 7 \ 6 \\
 3 \ 4 \\
 \hline
 1 \ 7 \ 1 \ 0 \ 4 \\
 1 \ 3 \ 2 \\
 1 \ 2 \ 8 \ 2 \ 8 \ 0 \\
 2 \ 1 \\
 \hline
 1 \ 4 \ 5 \ 3 \ 8 \ 4 \\
 \hline
 1
 \end{array}$$

Following on from Year 5... Column method for long multiplication

$$\begin{array}{r}
 1 \ 4 \ 2 \\
 3 \ 1 \\
 \hline
 1 \ 4 \ 2 \\
 4 \ 2 \ 6 \ 0 \\
 \hline
 1
 \end{array}$$

The introductory teaching of long multiplication, based on solid conceptual understanding of short multiplication, is developed in Years 4 and 5.

Children in Year 6 should be consolidating their understanding with the multiplication of increasingly large numbers, set in context wherever possible.

Column method for long multiplication, involving decimals

The multiplication of decimal numbers was introduced in year 5, with single digit numbers with up to 2 decimal places being multiplied by a single digit whole number.
(See Year 5 for accompanying notes)

$$\begin{array}{r}
 3 \ . \ 2 \ 5 \\
 6 \\
 \hline
 . \ 3 \ 0 \\
 1 \ . \ 2 \ 0 \\
 1 \ 8 \ . \ 0 \ 0 \\
 \hline
 1 \ 9 \ . \ 5 \ 0 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 3 \ . \ 2 \ 5 \\
 6 \\
 \hline
 1 \ 9 \ . \ 5 \ 0 \\
 \hline
 1 \ 1 \ 3
 \end{array}$$